



Lecture 13 - Wave excitation force

Observation and prediction

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Wave excitation force

Observation and prediction

August 31, 2015

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AALBORG UNIVERSITY
DENMARK

Agenda



Introduction

- Motivations

- Optimal control law

- MPC

Excitation force observers

- Soft sensor

Excitation force prediction



Introduction

Objectives

Introduction of different methodologies to observe and predict the wave excitation force.

¹What does "*best performance*" mean?



Introduction

Objectives

Introduction of different methodologies to observe and predict the wave excitation force.

Why there is a need to observe and predict the wave excitation force?

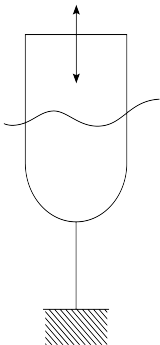
If we want to obtain the "*best performance*"¹ out of our system we need to know both the system and the system input.

Two examples are given below

¹What does "*best performance*" mean?

Introduction

RECAP



For a single dof, wave activated body WEC, i.e. heaving buoy or Wavestar WEC, the equation of motion can be expressed in frequency domain as: **(WARNING: sign convention)**

$$\left[i\omega(M + CM(\omega)) + CA(\omega) + \frac{K_{hst}}{i\omega} \right] V(\omega) = F_{ex}(\omega) + F_u(\omega) \quad (1)$$

Introducing the intrinsic mechanical impedence and substituting into the equation of motion, we obtain:

$$Z_i(\omega) = i\omega(M + CM(\omega)) + CA(\omega) + \frac{K_{hst}}{i\omega} \quad (2)$$

$$Z_i(\omega) V(\omega) = F_{ex}(\omega) + F_u(\omega) \quad (3)$$



Introduction

Optimal control

Optimal control law²

For a single dof wave activate body WEC, the optimal control law can be expressed in frequency domain as:

- ▶ 1 - optimal load (reactive or complex-conjugate control)

$$F_u(\omega) = -Z_i^*(\omega) V(\omega) \quad (4)$$

²Falnes, J. (2002). Ocean waves and oscillating systems: linear interactions including wave-energy extraction. Cambridge university press. Chapter 6.



Introduction

Optimal control

Optimal control law²

For a single dof wave activate body WEC, the optimal control law can be expressed in frequency domain as:

- ▶ 1 - optimal load (reactive or complex-conjugate control)

$$F_u(\omega) = -Z_i^*(\omega) V(\omega) \quad (4)$$

- ▶ 2 - optimal velocity (phase or amplitude control)

$$V_{opt}(\omega) = \frac{F_{ex}(\omega)}{2CA(\omega)} \quad (5)$$

²Falnes, J. (2002). Ocean waves and oscillating systems: linear interactions including wave-energy extraction. Cambridge university press. Chapter 6.

Introduction

Optimal control: Requirements



- In order to control the system we need to know the excitation force.

Introduction

Optimal control: Requirements



- ▶ In order to control the system we need to know the excitation force.
- ▶ The non-causality of the excitation force requires prediction of the incoming wave.

Introduction

Optimal control: Issues



- Both the optimal velocity and the optimal load defined by the optimal control theories are often unfeasible due to the system linearity. **don't try it in the lab unless the wave amplitude is very small!**

³Korde, U. (2000). Control system applications in wave energy conversion. In OCEANS 2000 MTS/IEEE Conference and Exhibition (Vol. 3, pp. 1817-1824). IEEE.



Introduction

Optimal control: Issues

- ▶ Both the optimal velocity and the optimal load defined by the optimal control theories are often unfeasible due to the system linearity. **don't try it in the lab unless the wave amplitude is very small!**
- ▶ In the complex-conjugate control the prediction of the system velocity is required because the *irf* is anticausal³. The prediction of a highly damped system is often unreliable (broad-banded response), therefore the phase control is a more robust choice. But in this case we need to include a velocity tracking control loop.

³Korde, U. (2000). Control system applications in wave energy conversion. In OCEANS 2000 MTS/IEEE Conference and Exhibition (Vol. 3, pp. 1817-1824). IEEE.



Introduction

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- ▶ The constraints are not easily implemented into the controller.

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Optimal control: Issues

- ▶ Both the optimal velocity and the optimal load defined by the optimal control theories are often unfeasible due to the system linearity. **don't try it in the lab unless the wave amplitude is very small!**
- ▶ In the complex-conjugate control the prediction of the system velocity is required because the *irf* is anticausal³. The prediction of a highly damped system is often unreliable (broad-banded response), therefore the phase control is a more robust choice. But in this case we need to include a velocity tracking control loop.
- ▶ The constraints are not easily implemented into the controller.
- ▶ **Can we still optimise the system taking care of the above issues?**

³Korde, U. (2000). Control system applications in wave energy conversion. In OCEANS 2000 MTS/IEEE Conference and Exhibition (Vol. 3, pp. 1817-1824). IEEE.

Introduction

MPC



Model
Predictive
Control

Model predictive control optimises the control trajectory, given a dynamic model of the system and its constraint(s)

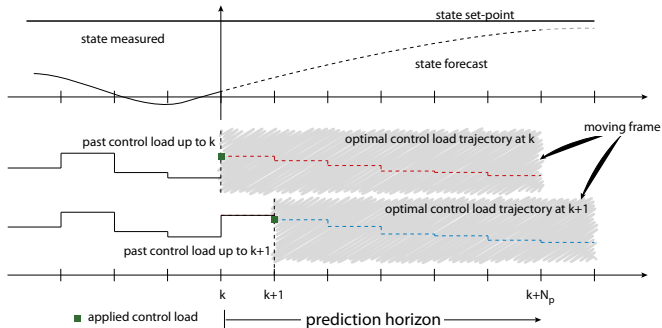
Introduction

MPC

Model
Predictive
Control

Model predictive control optimises the control trajectory, given a dynamic model of the system and its constraint(s)

MPC working principle



Introduction

MPC: Reference list



We will not go through the formulation details; anybody is welcome to ask question though.

- ▶ Gieske, P. (2007). Model predictive control of a wave energy converter: Archimedes wave swing. Delft University of Technology, Delft, The Netherlands.
- ▶ Cretel, J. A., Lightbody, G., Thomas, G. P., and Lewis, A. W. (2011, September). Maximisation of energy capture by a wave-energy point absorber using model predictive control. In Proceedings of the 18th IFAC World Congress, Milano, Italy, Aug (pp. 3714-3721).
- ▶ Brekken, T. K. (2011, June). On model predictive control for a point absorber wave energy converter. In PowerTech, 2011 IEEE Trondheim (pp. 1-8). IEEE.
- ▶ Hals, J., Falnes, J., and Moan, T. (2011). Constrained optimal control of a heaving buoy wave-energy converter. Journal of Offshore Mechanics and Arctic Engineering, 133(1), 011401.

Introduction

MPC: Pros and Cons



Cons (starting from the darkside):

The following points are given in comparison with a simpler PI controller

- ▶ High formulation complexity

Introduction

MPC: Pros and Cons



Cons (starting from the darkside):

The following points are given in comparison with a simpler PI controller

- ▶ High formulation complexity
- ▶ Relative high computational cost: this can be a serious issue for non-linear MPC

Introduction

MPC: Pros and Cons



Cons (starting from the darkside):

The following points are given in comparison with a simpler PI controller

- ▶ High formulation complexity
- ▶ Relative high computational cost: this can be a serious issue for non-linear MPC
- ▶ Requires to forecast the system state and the system inputs (disturbances)

Introduction

MPC: Pros and Cons



Pros:

- ▶ The constraints are embedded in the formulation of the minimisation problem.

Introduction

MPC: Pros and Cons



Pros:

- ▶ The constraints are embedded in the formulation of the minimisation problem.
- ▶ Customisable cost function

Introduction

MPC: Pros and Cons



Pros:

- ▶ The constraints are embedded in the formulation of the minimisation problem.
- ▶ Customisable cost function
- ▶ Higher performance (based on numerical simulation)

MPC

Summary



The knowledge of the excitation force is still required.
Further, the MPC requires the prediction of the excitation force over the prediction horizon.

Excitation force observers

Introduction



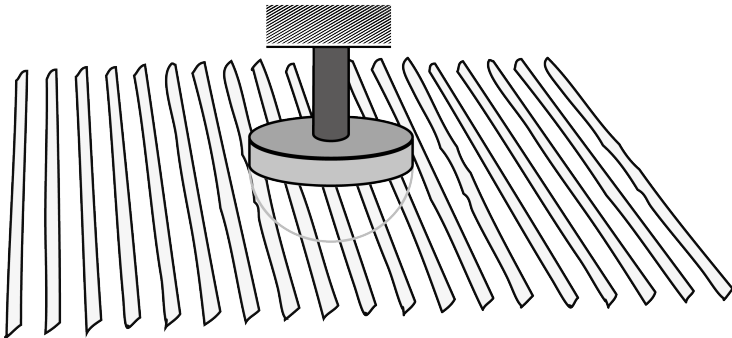
Before being able to predict the excitation force we need to measure it, but how?

Excitation force observers

Introduction

Before being able to predict the excitation force we need to measure it, but how?

Remember how the excitation force is defined



Excitation force observers

Soft sensors



13

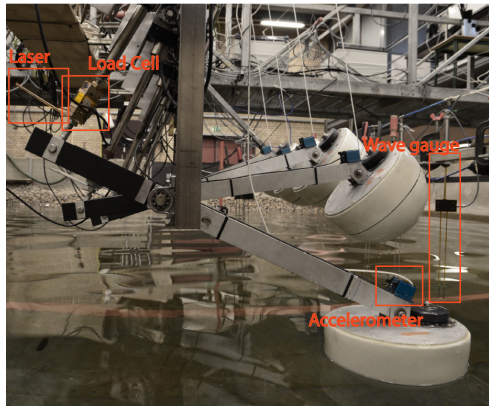
Combining measurable variables it is possible to obtain an estimation of the excitation force (observe).

Excitation force observers

Soft sensors

Which are the (commonly) measurable variables?

- 1 - Surface elevation
- 2 - System state
- 3 - Loads





Excitation force observers

Soft sensors

Sea surface elevation

Short term wave forecasting and excitation force observer using FIR/IIR filter ⁴.

- ▶ Wave prediction based on measurement up-wave (????) and wave model. FIR/IIR filters based on analytical or fitted models, i.e. wave propagation model, Auto Regressive models, Neural Networks, etc.

⁴Ferri, F., Sichani, M. T., and Frigaard, P. (2012, January). A Case Study of Short-Term Wave Forecasting Based on FIR Filter: Optimization of the Power Production for the Wavestar Device. In The Twenty-second International Offshore and Polar Engineering Conference. International Society of Offshore and Polar Engineers.

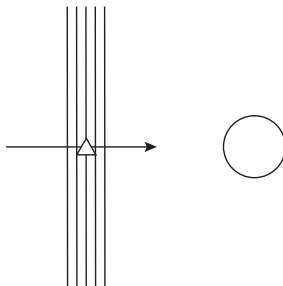
Excitation force observers

Soft sensors

Sea surface elevation

Short term wave forecasting and excitation force observer using FIR/IIR filter⁴.

Wave prediction
wave model. FIR
i.e. wave propagation
Networks, etc.



:(????) and
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- ▶ Convolution of the non-causal wave excitation force *irf* with the predicted sea surface at the floater location.

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- ▶ Convolution of the non-causal wave excitation force *irf* with the predicted sea surface at the floater location.
- ▶ **The prediction of the sea surface in short crested sea states can be cumbersome!**

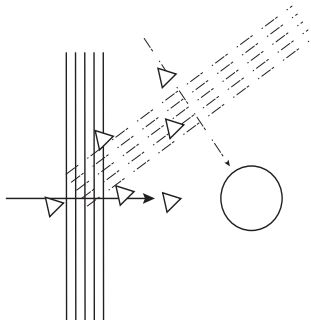
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Excitation force observers

Soft sensors

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Excitation force observers

Soft sensors



System state and applied load

Using the system state, the applied load and a model of the system is possible to assess the system input.



Excitation force observers

Soft sensors

System state and applied load

Using the system state, the applied load and a model of the system is possible to assess the system input.

Starting from the equation of motion, the excitation force can be obtained as.

$$f_{ex}[k] = f_u[k] - f_{INERTIA}(\dot{v}(t)) - f_{RAD}(v(t))[k] - f_{hst}(p(t))[k] \quad (6)$$

here $[k]$ represents the actual instant of time.



Excitation force observers

Soft sensors

System state and applied load

Using the system state, the applied load and a model of the system is possible to assess the system input.

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here $[k]$ represents the actual instant of time.

LIMITATIONS: Any error in the measurement and in the model are included in the excitation force. We miss a feedback from the system state.

Excitation force observers

Soft sensors



System state and applied load, alternatives

It is possible to use the information of the system state to evaluate the model error and then correct the excitation force assessed.

1. Luenberger observer
2. Kalman filter



Excitation force observers

Soft sensors

Luenberger observer

Given a dynamic model of a system

$$x[k + 1] = Ax[k] + Bu[k] \quad (7)$$

$$y[k] = Cx[k] \quad (8)$$

Excitation force observers

Soft sensors

Luenberger observer

Given a dynamic model of a system

$$x[k + 1] = Ax[k] + Bu[k] \quad (7)$$

$$y[k] = Cx[k] \quad (8)$$

If the system is observable, then it is possible to identify a matrix L such that the error between the plant (x) and the observed state (\hat{x}) tends to zero.

$$\hat{x}[k + 1] = A\hat{x}[k] + L(y[k] - C\hat{x}[k]) + Bu[k] \quad (9)$$

$$e[k + 1] = (A - LC)e[k] \quad (10)$$

The error dynamic can be chosen by varying the matrix $A - LC$

Excitation force observers

Soft sensors



Luenberger observer

How does the dynamic model of a WEC look like?

Excitation force observers

Soft sensors

Luenberger observer

How does the dynamic model of a WEC look like?

Since we want to estimate the excitation force, we need to expand the dynamic model of the system and include the excitation force.

$$A = \left[\begin{array}{cc|c|c} 0 & 1 & \underline{0} & 0 \\ -\frac{k}{J} & -\frac{d_R}{J} & -\frac{c_R}{J} & B_W \cdot C_{ex} \\ \hline \underline{0} & b_R & a_R & \underline{0} \\ \hline \underline{0} & \underline{0} & \underline{0} & A_{ex} \end{array} \right], B = \left[\begin{array}{c} 0 \\ \frac{1}{J} \\ \hline 0 \\ \hline 0 \end{array} \right], C = \left[\begin{array}{cc|c|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]$$

where the state vector is defined as:

$$x[k] = \left[\begin{array}{cc|c|c} p & v & x_R & f_{ex} \end{array} \right]^T$$

Excitation force observers

Soft sensors

Luenberger observer

The excitation model is a simple integrator therefore

$$A_{ex} = 1, \quad B_{ex} = 0, \quad C_{ex} = 1$$

Other models are possible, such as oscillator observer.

Excitation force observers

Soft sensors

Luenberger observer

The excitation model is a simple integrator therefore

$$A_{ex} = 1, \quad B_{ex} = 0, \quad C_{ex} = 1$$

Other models are possible, such as oscillator observer.

The L matrix is defined as

$$L = PC^T R_L^{-1}$$

where P is obtained by solving the Riccati equation of the system

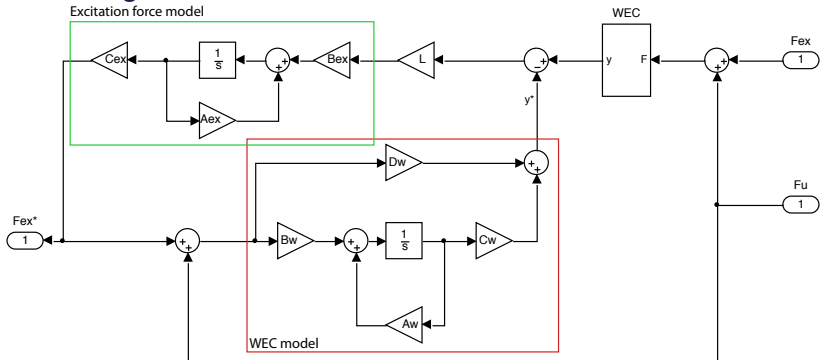
$$AP + PA^T - PC^T R_L^{-1} CP + Q_L = 0$$

Q_L and R_L are the covariance matrices of the process and the measurement (tuning parameters)

Excitation force observers

Soft sensors

Luenberger observer



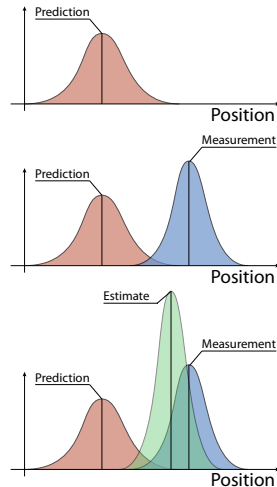
Excitation force observers

Soft sensors

Kalman filter

Kalman filter provide the best linear estimation of the system state.

It uses a statistical representation of the system and combine the system prediction with its measurements.



Excitation force observers

Soft sensors

Kalman filter

Kalman filter breakout:

- ▶ Prediction - a priori estimate
 - ▶ $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_k$
 - ▶ $P_{k|k-1} = AP_{k-1|k-1}A^T + Q_k$

Excitation force observers

Soft sensors

Kalman filter

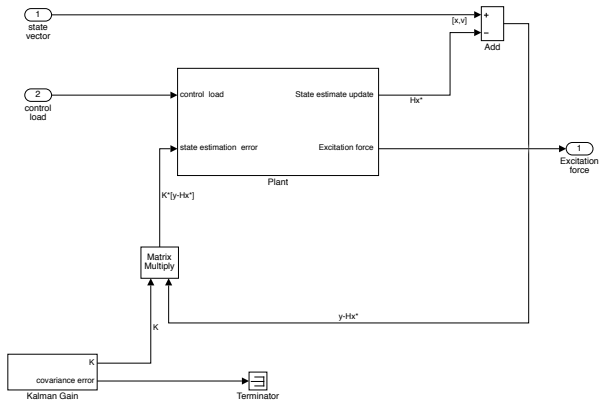
Kalman filter breakout:

- ▶ Prediction - a priori estimate
 - ▶ $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_k$
 - ▶ $P_{k|k-1} = AP_{k-1|k-1}A^T + Q_k$
- ▶ Update - a posteriori estimate
 - ▶ $\tilde{y}_k = y_k - C\hat{x}_{k|k-1}$
 - ▶ $S_k = CP_{k|k-1}C^T + R_k$
 - ▶ $K_k = P_{k|k-1}C^TS_k^{-1}$
 - ▶ $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\tilde{y}_k$
 - ▶ $P_{k|k} = (I - K_kC)P_{k|k-1}$

Excitation force observers

Soft sensors

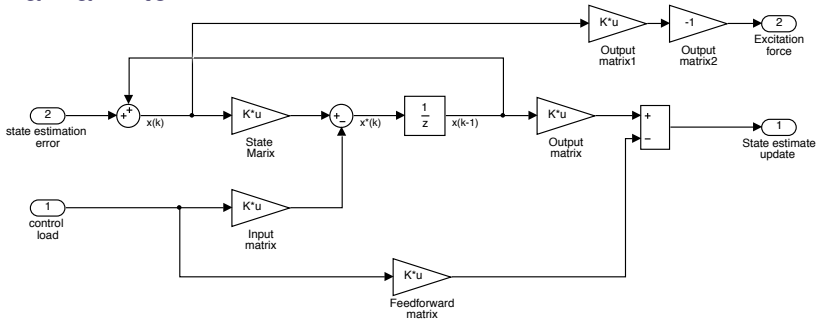
Kalman filter



Excitation force observers

Soft sensors

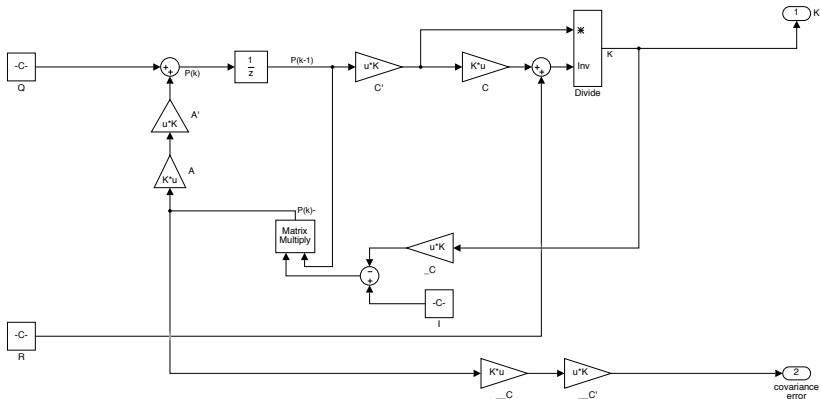
Kalman filter



Excitation force observers

Soft sensors

Kalman filter





Excitation force prediction

Introduction

Once the excitation force is known it is possible to predict its evolution in time. Excitation force models:

- ▶ Autoregressive Model
- ▶ Cyclical Model
- ▶ Cyclical Model with variable frequency

Alternatives

Nerural network and Fuzzy Logic are also viable solutions

Excitation force prediction

Autoregressive Model

Autoregressive (AR) model assumptions:
the variable can be predicted using a linear combination of the past value of the variable

$$\hat{f}_{ex}[k+1|k] = \sum_{i=0}^{N-1} a_i \cdot f_{ex}[k-i]$$



Excitation force prediction

Cyclical Model

Cyclical model assumptions:

the excitation force is expressed as a superposition of a number m of linear harmonic components. The choice of m and the distribution of the harmonics within the wave spectrum is a key point.

Excitation force prediction

Cyclical Model

Cyclical model assumptions:

the excitation force is expressed as a superposition of a number m of linear harmonic components. The choice of m and the distribution of the harmonics within the wave spectrum is a key point.

Assuming the index i ranging from 1 to m the model can be expressed as:

$$\begin{bmatrix} \psi_i[k+1] \\ \psi_i^*[k+1] \end{bmatrix} = \begin{bmatrix} \cos(w_i \Delta T) & \sin(w_i \Delta T) \\ -\sin(w_i \Delta T) & \cos(w_i \Delta T) \end{bmatrix} \begin{bmatrix} \psi_i[k] \\ \psi_i^*[k] \end{bmatrix} + \begin{bmatrix} \xi_i[k] \\ \xi_i^*[k] \end{bmatrix} \quad (11)$$

$$f_{ex}[k] = \sum_{i=1}^m \psi_i[k] + \zeta[k] \quad (12)$$

Excitation force prediction

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$$f_{ex}[k] = \sum_{i=1}^m \psi_i[k] + \zeta[k] \quad (12)$$

The best estimation of $\hat{f}_{ex}[k|k]$ is obtained using a Kalman filter, while the N-step ahead prediction is achieved from the free-evolution of the dynamical model ($\hat{f}_{ex}[k+N|k] = CA^N \hat{x}[k|k]$).



Excitation force prediction

Cyclical Model with variable frequency

Cyclical model with variable frequency assumptions:
the excitation force is expressed as a superposition an harmonic components, which frequency is a function of time. This eliminate the error in the placement of the harmonic components but generate a non-linear system.

Excitation force prediction

Cyclical Model with variable frequency

Cyclical model with variable frequency assumptions:

the excitation force is expressed as a superposition an harmonic components, which frequency is a function of time. This eliminate the error in the placement of the harmonic components but generate a non-linear system.

The excitation force is now defined by a single cyclical model as:

$$\begin{bmatrix} \psi_i[k+1] \\ \psi_i^*[k+1] \\ \omega[k+1] \end{bmatrix} = \begin{bmatrix} \cos(\omega[k]\Delta T) & \sin(\omega[k]\Delta T) & 0 \\ -\sin(\omega[k]\Delta T) & \cos(\omega[k]\Delta T) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_i[k] \\ \psi_i^*[k] \\ \omega[k] \end{bmatrix} + \begin{bmatrix} \xi_i[k] \\ \xi_i^*[k] \\ \kappa[k] \end{bmatrix} \quad (13)$$

$$f_{ex}[k] = \psi_i[k] + \zeta[k] \quad (14)$$

Excitation force prediction

Cyclical Model with variable frequency

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$$f_{ex}[k] = \psi_i[k] + \zeta[k] \quad (14)$$

Since the model is non-linear an Extended-Kalman filter can be used to obtain the excitation force estimation.

Excitation force prediction

Side notes



Search for Fusco F, for a number of publications over this matter.

TIP: The prediction methods proposed have a GOF below 50%. In order to increase the number it is possible to low pass filter the signal. Indeed we are interested mostly in the prediction of the low frequency component of the spectrum.



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